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Boethius' Contribution to the *Quadrivium*. Selected bibliography

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Bibliography

1. Barrett, Sam. 2013. *The Melodic Tradition of Boethius' De consolatione philosophiae in the Middle Ages*. Kassel: Bärenreiter.
Volume I: Text; Volume II: Transcriptions and Commentary.

2. Bell, Nicolas. 2009. "Readings and Interpretations of Boethius's *De institutione musica* in the Later Middle Ages." In *Sapientia et Eloquentia: Meaning and Function in Liturgical Poetry, Music, Drama, and Biblical Commentary in the Middle Ages*, edited by Iversen, Gunila and Bell, Nicolas, 365-380. Turnhout: Brepols.
"Throughout the Middle Ages the primary text for the understanding of music was Boethius's *De institutione musica*. Boethius was essentially a philosopher. The purpose of his discourses in the liberal arts was to enable the mind to move closer to an abstract truth: it was only after one understood the truth that was inherent in music, in arithmetic, or in the other liberal arts, that one could begin to have an understanding of philosophy.(1) He was also a theologian — or at least, as far as Boethius was concerned, the purpose of the study of music was essentially a theological one. In the *De institutione musica*, the extensive discussions of instrumental music were intended not simply as an exemplification of the geometrical principles of harmonic theory; they were also the first stage in an intellectual progression, first from *musica instrumentalis* to *musica humana* — from the practical understanding of the monochord to the understanding of the human soul — and ultimately from *musica humana* to *musica mundana* — to the divine plan of the world and the heavens, the music of the spheres." (p. 365)
(1) See Calvin M. Bower, 'The Role of Boethius' De institutione musica in the Speculative Tradition of Western Musical Thought', in *Boethius and the Liberal Arts: A Collection of Essays*, ed. by Michael Masi, Utah Studies in Literature and Linguistics, 18 (Bern: Lang, 1981), pp. 157–74.

3. Bernhard, Michael. 1990. "Glosses on Boethius' "De institutione musica"." In *Music Theory and its Sources: Antiquity and the Middle Ages*, edited by Barbera, André, 136-149. Notre Dame: University of Notre Dame Press.

4. Bohlin, Erik. 2017. "An Unnoticed Gloss in Boethius, *Institutio arithmetica* II, 54." *Latomus* no. 76:403-408.
"The last chapter of the second book of Boethius' Institutio arithmetica deals with the proportion which in the chapter's heading is called maxima et perfecta symphonia "the greatest perfect concord". With modern notation the proportion can be described thus: In a sequence of four numbers a, b, c, d – which are positive integers and for which a > b > c > d – the proportion is a : c = b : d, or, inversely, c : a = d : b, and such that b = (a + d) / 2 and c = (2ad) / (a + d).
The numbers d, b, a will thereby constitute an arithmetic progression, in which b is the arithmetic mean, and the numbers d, c, a a harmonic progression, in which c is the harmonic mean. The sequence is exemplified and expounded further in the chapter by the numbers 12 (= a), 9 (= b), 8 (= c), and 6 (= d)."
(...)
"According to the editions of G. Friedlein (1867), J.-Y. Guillaumin (1995), and H. Oosthout and J. Schilling (1999), the passage reads thus:
In quattuor enim terminis si fuerit quemadmodum primus ad tertium, sic secundus ad quartum, proportionum ratione scilicet custodita, geometrica medietas explicatur et quod continetur sub extremitatibus aequum erit ei quod sub utraque medietate ad se inuicem multiplicata conficitur. (Boeth., arithm. II, 54, 3) 4
Guillaumin translates it into French as follows:
"En effet, dans quatre termes [a, b, c, d], si le second [b] est au quatrième [d] comme le premier [a] est au troisième [c], la raison des rapports se conservant, cela développe une médiété géométrique et le produit des extrêmes [ad] sera égal au produit des deux moyens multipliés entre eux [bc]." (5)
I shall now demonstrate that the words *ad se inuicem multiplicata* are redundant and most likely spurious. (pp. 403-404, notes omitted)

(...)
"To conclude: As it has been demonstrated (i) that the words *ad se inuicem multiplicata* are superfluous at *arithm*. II, 54, 3 and (ii) that there are no attestations in Boethius' work to indicate that he uses such phrases with the participle *multiplicatus* in a redundant fashion, I propose that the words in question are not of Boethius' hand, but have been added and inserted into the text as a gloss." (p. 408)
(5) J.-Y. Guillaumin, Boèce. *Institution arithmétique*, Paris, 1995, p. 175

5. Bower, Calvin. 1984. "The Modes of Boethius." *The Journal of Musicology* no. 3:252-263.

6. Bower, Calvin M. 1978. "Boethius and Nichomachus: An Essay Concerning the Sources of *De institutione musica*." *Vivarium* no. 16:1-45.
"I begin this inquiry with two underlying assumptions. First, Boethius was principally a translator when putting together the *De institutione musica*. The treatise seems to follow the *De institutione arithmetica* in the chronology of Boethius' works, and the arithmetical treatise is recognized to be a translation of Nicomachus of Gerasa εισαγωγή άριθμητική.
(...)
Boethius' method of composing in his early works is that of compiling through translation with some commentary; the arithmetical treatise and the logical works clearly demonstrate this point. Thus ininquiring into Boethius' sources I am trying to determine which Greek treatise Boethius was translating when he compiled his musical treatise.
My second assumption is that Boethius was a conscientious and competent translator." (pp. 1-2)
(...)
"Since this essay is somewhat expository in nature, its organization must largely follow that of Boethius' treatise. Seven principal sections will be designated as follows:
I. Pattern of citation in the mathematical works
II. Books I and II
III. Book III
IV. Book IV
V. Unity of Books I-IV
VI. Nicomachus and Ptolemy
VII. Book V and the original scope of *De institutione musica*
In that my conclusions concerning the first three books are similar to those of Pizzani (*) and other writers, these sections may be brief and concise. Since my treatment of Book IV stands in sharp contrast to previous scholarship, that section must be the most detailed and extended." (pp. 3-4 notes omitted)
(*) Ubaldo Pizzani 'Studi sulle fonti del " De Institutione Musica" di Boezio', Sacris erudiri, 16 (1965), 5-164.

7. ———. 1981. "The Role of Boethius' *De Institutione Musica* in the Speculative Tradition of Western Muiscal Thought." In *Boethius and the Liberal Arts: A Collection of Essays*, edited by Masi, Michael, 157-174. Bern: Peter Lang.
"An attempt to define the role of Boethius' *De institutione musica* in the speculative tradition of Western musical thought may appear to be an awesome and even pretentious task, especially in context of a study as brief as the present one. My limitations may seem even more severe in that I will confine my discussion to musical writings before the year 1100. The centuries immediately prior to 1100 saw the birth of that sphere of study which has come to be called musical theory; the nature of the thought which has grown and developed within this sphere was largely shaped during the years between about 500, when Boethius compiled his *De institutione musica*, and the year 1000, when the first full flowering of medieval musical theory was completed in the works of such theorists as Guido of Arezzo and Herman of Reichenau.

If my temporal and spatial boundaries for this study seem small, I would like my consideration of the word "speculative" to seem large. By "speculative tradition" I do not mean what Boethius would term *musica mundana* or even *musica humana*; I mean rather man's verbal reflections and meditations concerning an art so universal yet so difficult to grasp intellectually and articulate verbally. If my study has any basic thesis, it is that Boethius' *De institutione musica* played a highly significant and clearly definable role in shaping the language and concepts with which Western man sought to understand and articulate music. One might begin to prove this thesis by compiling an index of the places in musical theory where Boethius has been quoted or cited, or those relatively few places where he has been both quoted and cited. But such an approach seems too discursive; moreover, it has been basically accomplished in several other studies. (1) The mere quoting of an author does not prove that his thought is crucial in shaping the ideas of the writer quoting, especially during the Middle Ages. My approach will be to examine Boethius' basic position and attitude toward music, both in itself and in comparison with other theorists. Thereafter I will trace the history of Boethius' text and of reflections concerning music in subsequent centuries. Finally, I will show how Boethius' attitude essentially shaped the understanding of and articulation concerning music in the ninth through the eleventh centuries.
In discussing the nature of Boethius' treatise I do so with the understanding that it is basically a translation of Greek sources and not an original work. (2) But for the sake of ease in discourse, as well as the fact that medieval man viewed the work as the creation of Boethius, I shall speak of the basic contents of the work as words and thoughts of Boethius." (p. 157)
(1) W. Brambach, *Die Musiklitteratur des Mittelalters bis zur Blüthe der Reichenauer Sängerschule* (Karlsruhe, 1883). Gerhard Pietzsch, *Die Klassifikation der Musik von Boetius bis Ugolino von Orvieto* (Halle, 1929).
(2) Ubaldo Pizzani, "Studi sulle fonti del *De Institutione Musica* di Boezio," *Sacris Erudiri* XVI (1965), 5-160; C.M. Bower, "Boethius and Nicomachus: An Essay Concerning The Sources of *De institutione musica*," *Vivarium* XVI (1978), 1-45.

8. ———. 2002. "The Transmission of Ancient Music Theory into the Middle Ages." In *The Cambridge History of Western Music Theory*, edited by Christyensen, Thomas, 136-167. Cambridge: Cambridge University Press.
"Boethius, following Pythagorean and neo-Platonic authors before him, held that quantity was divided into two basic genera: discrete quantity – or multitude; and continuous quantity – or magnitude. The *monad*, or unity, was the source of discrete quantity, and this genus could increase into infinite multitude; yet its basic element, unity, remained indivisible. Magnitude, or continuous quantity, might be represented by the line or a shape, which was delimited with respect to increasing and growth, but could be infinitely divided. The two basic genera of quantity were, in turn, subdivided into two species: multitude is best represented by number, and every number can be considered in and of itself (even, odd, perfect, square, cube, etc), or it can be considered in relation to another (in ratios and proportions – e.g., 2:1, 3:2, or 6:4:2); magnitude is best represented by shapes, and some shapes are fixed and immobile (e.g., a line, a triangle, a cube), while others are in motion (e.g., the sun, the moon, the heavenly spheres). Four areas of study were thus defined by the very nature of quantity: *arithmetic* pursued number in and of itself; *music* examined number in ratios and proportions; *geometry* considered immobile magnitudes; *astronomy* investigated magnitudes in motion. Boethius described these four disciplines as the *quadrivium*, the fourfold path by which the soul was led from the slavery of sensual knowledge to the mastery of knowing immutable essences. *Musica* thus became a necessary prerequisite to the study of philosophy. (19)" (pp. 141-142)
(19) See *De institutione arithmetica* I.1; *De institutione musica* II.3

9. Caiazzo, Irene. 2020. "Medieval Commentaries on Boethius's *De arithmetica*: A Provisional Handlist." *Bulletin de philosophie médiévale* no. 62:3-13.

10. Caldwell, John. 1981. "The *De Institutione Arithmetica* and the *De Institutione Musica*." In *Boethius: His Life, Thought and Influence*, edited by Gibson, Margaret, 135-154. Oxford: Blackwell.
"Boethius stands at the very end of a long tradition of mathematical philosophy that had extended for nearly a thousand years. (1) His work on music is much the more original of the two treatises to be considered in this chapter, and it is to that that the greater part of our attention will be devoted; but Boethius also wrote on arithmetic at least of the other mathematical arts, and his contribution must be considered in the light of a tendency in antiquity to isolate the four speculative mathematical subjects — arithmetic, music, geometry and astronomy — and to group them together as a unit which Boethius was apparently the first to call the 'quadrivium' or 'quadruvium'. (2) Opposed to this of course was the trivium of grammar, rhetoric and dialectic, the whole forming the group of sevenu liberal arts' which combined with the 'three philosophies' made up the staple diet of the medieval university curriculum. With these larger units we are not here concerned. But the development of the quadrivium, and the contribution of Boethius to it, demands a closer look." (p. 135)
(1) In the East, however, writers such as Michael Psellus, Bryennios and Pachymeres reproduced ancient musical theory in their treatises of the Byzantine middle ages, just as Boethian philosophy is reproduced in the works of Hucbald, Regino of Prüm, Johannes de Mûris and innumerable others of the West. But even if Boethius himself were proved to be a wholly unoriginal thinker in music, his work would still belong to the very end of the long cultural tradition to which it relates, and from which the works of Cassiodorus and Isidore already mark a significant break.
(2) *Arith*. I. i: pp. 7. 25, 9. 28.

11. Chamberlain, David S. 1970. "Philosophy of Music in the *Consolatio* of Boethius." *Speculum* no. 45:80-97.
Reprinted in Manfred Fuhrmann und Joachim Gruber (Hrsg.), *Boethius*, Darmstadt: Wissenschaftliche Buchgesellschaft, 1984, pp. 377–404.

12. Courcelle, Pierre. 1981. "Boethius, Lady Philosophy, and the Representations of the Muses." In *Boethius and the Liberal Arts: A Collection of Essays*, edited by Masi, Michael, 211-218. Bern: Peter Lang.

13. Dehnert, Edmund John. 1969. "Music as Liberal in Augustine and Boethius." In *Arts Libéraux et Philosophie Au Moyen Âge. Actes du Quatrième Congrès International de Philosophie Médiévale*, 987-991. Paris: Vrin.

14. Edmiston, Jean. 1974. "Boethius on Pythagorean Music." *Music Review* no. 35:179-184.

15. Erickson, Raymond. 1992. "Eriugena, Boethius, and the Neoplatonism of *Musica* and *Scolica Enchiriadis*." In *Musical Humanism and Its Legacy: Essays in Honor of Claude V. Palisca*, edited by Kovaleff Baker, Nancy and Russano Hanning, Barbara, 53-78. Stuyvesant, NY: Pendragon Press.
"Because Eriugena's writings include discussions of the liberal arts and music, they have attracted the interest of music historians. And because Eriugena uses certain terms that are found in the *Enchiriadis* treatises, scholars for the past 150 years have argued for, or at least accepted the likelihood of, links between Eriugena and the treatises. Indeed, no less eminent a scholar than Jacques Handschin was certain that Eriugena knew *Musica enchiriadis*. (8) Furthermore, none of today's leading authorities on Carolingian theory — even when a proposed point of contact has been disputed — has argued against such links altogether. (9) It is the intention here, however, to do just that: to assert that there is no likely connection between Eriugena and the *Enchiriadis* treatises and to demonstrate that the concepts and vocabulary of the *Enchiriadis* treatises that have been attributed to Eriugena (and indirectly to Greek Neoplatonism) are all explainable in terms of the earlier Latin tradition, and Boethius in particular, to that end, we will undertake a brief review of

previous scholarship on this question and a more detailed critique of new, as yet unevaluated claims for Eriugenian influence." (pp. 56-57)
(8) “Die Musikanschauung des Johannes Scotus (Erigena),” Deutsche Vierteljahrsschrift für Literaturwissenschaft und Geistesgeschichte V (1927) 339.
(9) For example, Michel Huglo, “Bibliographie des éditions et études relatives â la théorie musicale du Moyen Age," Acta musicologica LX (1988) 261, refers to the author of *Musica enchiriadis* as “disciple de Jean Scot”."

16. Evans, Gillian R. 1975. "The influence of quadrivium studies in the eleventh. and twelfth-century schools." *Journal of Medieval History* no. 1:151-164.
Abstract: "This p·aper considers the nature of some of the evidence for the study ef the subjects of the quadrivium in the eleventh and twelfth centuries, and the sources of contemporary interest in the mathematical arts. The survival in the eleventh and twelfth centuries qf manuscripts ef the quadrivium texts of such late-classical writers as Boethius and Martianus Capella is considered, and their mathematical ideas are analysed in relation particularly to the thought of William of Conches, Peter Abelard and others. An examination ef mathematical notions employed in the specific context ef textbook commentary on Aristotle's *Categories* is followed by discussion of the method of proving the unity of the Trinifv by means of analogy with the properties of points, lines and surfaces."

17. ———. 1978. "Introductions to Boethius's “Arithmetica” of the Tenth to the Fourteenth Century." *History of Science* no. 16:22-41.
"The much-neglected introductions to arithmetical texts — especially Boethius’s *Arithmetica* — which are to be found in a number of manuscripts of the tenth to the fourteenth century, are essentially teaching-aids. Even if they were not necessarily used on every occasion in the actual presence of a master, they preserved his spoken comments for the use of the student reading alone. In whatever way they were employed, their purpose is primarily instructional; they complement the text to be studied, and they seem to have no pretensions to stand as works of literature in their own right. It is striking that the authors of many of these introductions, commentaries and glosses have often found it convenient to explain arithmetic by means of notions drawn from grammar, logic or rhetoric. Grammar and logic in particular were subjects undergoing active development during these centuries." (p. 22)

18. ———. 1978. "A commentary on Boethius's *Arithmetica* of the twelfth or thirteenth century." *Annals of Science* no. 35:131-141.

19. ———. 1981. "Boethius’ *Geometry* and the four Ways." *Centaurus* no. 25:161-165.
"It is evident on the most cursory reading that neither of the *Geometries* which have survived under Boethius’ name are, as they stand, authentic works. They contain material which places them, at the earliest, in the tenth century; they are disorderly and repetitious. Nevertheless, there is every reason to believe that Boethius wrote a *Geometria*, and it is not, on the face of it, impossible that some fragments of the original may survive, embedded in these fraudulent treatises. Some work has been done upon the Euclidean translations which are found in both versions, but little attempt has been made to answer the question: what should we expect the *Geometria* to have contained? Boethius himself provides some evidence on this point which has been largely overlooked." (p. 161)

20. ———. 1994. "John of Salisbury and Boethius on arithmetic." In *The World of John of Salisbury*, edited by Wilks, Michaerl, 161-167. Oxford: Blackwell.

21. Folkerts, Menso. 1981. "The Importance of the Pseudo-Boethian *Geometria* during the Middle Age." In *Boethius and the Liberal Arts: A Collection of Essays*, edited by Masi, Michael, 187-209. Bern: Peter Lang.
Reprinted as Essay VII in M. Folkerts, *Essays on Early Medieval Mathematics: The Latin Tradition*, New York: Routledge 2003.
"Compared to the other writings of Boethius on the *Trivium* and *Quadrivium*, his *Geometria* takes a special place. We do not have this work in its original form, but

only in two later adaptations: both contain only part of the original but on the other hand they are enlarged through a variety of insertions. Therefore any study of the importance of Boethius' *Geometria* in the Middle Ages should not only try to show the influence of geometrical writings which were transmitted under the name of Boethius, but also should try to understand the origin of such compilations. From these two demands, then, arises the organization of my essay: the first section will be concerned with the scanty evidence known about the authentic *Geometria* as well as with the contents and origins of both extant compilations. In the second part of my essay I will attempt to show the dissemination and impact of both writings during the Middle Ages. For this purpose I will make use of entries of the *Geometria* in medieval library catalogues and of allusions to it in other medieval writings. A broader understanding of the importance of these two writings and of their comprehension in this period can be gained from an analysis of the scholia to one of these compilations which have been neglected up to now. Such a study should make it possible to indicate the value of these treatises in comparison with the other two, authentic, works of the Boethian quadrivium - the *Arithmetica* and *Musica.*" (p. 187)

22. ———. 2003. "The Geometry II Ascribed to Boethius." In *Essays on Early Medieval Mathematics: The Latin Tradition*, edited by Folkerts, Menso. New York: Routledge.
Essay IX, This is the English translation, with some changes, of the article "Das Problem der pseudo-boethischen Geometrie", *Sudhoffs Archiv* 52 (1968), 152-161. A more detailed analysis is given in Folkerts, *"Boethius" Geometrie II, ein mathematisches Lehrbuch des Mittelalters*, Wiesbaden 1970.

23. Fournier, Michael. 2008. "Boethius and the consolation of the *quadrivium*." *Mediaevalia et Humanistica* no. 34:1-21.
"Conclusion. The books of the *Consolation* form a step-by-step ascent from the lower part of the soul to the higher. For Boethius, the ascent passes from sense (book 1) and imagination (book 2) to reason (books 3–4), but it ends with a glimpse of what is beyond reason (book 5). Intellect and not reason characterizes the divine life and is its mode of knowing all the lower modes in a simple way. Boethius moves the reader from the lower modes of knowing to the higher by showing the limits of each mode. At the end of each book there is an opposition or a contradiction that cannot be resolved by the mode and points to the need to adopt a higher mode of knowing.
The *Consolation* not only advances from lower modes of knowing to higher, but also presents an ascent through the levels of being. When Philosophy appears to the Prisoner her height is ambiguous (*Consolation* 1,1,1), and she appears at one moment to "confine herself to the ordinary measure of man" (*Consolation* 1,1,2), while at another moment it appears that "the crown of her head touched the heavens" (*Consolation* 1,1,2) and at yet another she appears to have "penetrated the heavens themselves" (*Consolation* 1,1,2) and passed beyond the reach of human vision. The three heights of Lady Philosophy represent the terrestrial world of process, the spheres of the heavens that circumscribe the natural world, and the transcendent divinity. For Boethius, consolation requires an ascent from the lower, human perspective to the highest, divine perspective, even if this divine perspective is only intimated or adumbrated." (p. 19)

24. Guillaumin, Jean-Yves. 2012. "Boethius's *De institutione arithmetica* and its Influence on Posterity." In *A Companion to Boethius in the Middle Ages*, edited by Kaylor Jr., Noel Harold and Phillips, Philip Edward, 135-161. Leiden: Brill.
Abstract: "The first systematic and well-developed treatise on the mathematical subject in the Roman world, the *De institutione arithmetica* should be considered as the first fruits of the intellectual activity of the young Boethius. It is from the pen of Boethius in the *De institutione arithmetica* that we find the first use of the word *quadriuium* to designate the meeting of the four mathematical sciences: arithmetic, music, geometry, and astronomy. Boethius was intrigued at least as much by the

philosophical aspect of the study of arithmetic as by its strictly technical and scientific aspects in the modern sense of the term. Without a doubt, what Boethius and Nicomachus understood by arithmetic would be much better expressed today by the term arithmology. The arithmetic treatise of Boethius profoundly marked the intellectual evolution of the West during the High Middle Ages and up to the beginning of the Renaissance."

25. Heilmann, Anja. 2013. "'amica est... similitudo' (Boeth. mus. 1,1): Musiktheorie und musikalische Ästhetik bei Boethius." In *Das Schöne in Theologie, Philosophie und Musik*, edited by Mayer, Cornelius, Müller, Christof and Förster, Guntram, 83-94. Würzburg: Echter Verlag.

26. Hicks, Andrew. 2016. "Re-interpreting an Arithmetical Error in Boethius's *De institutione musica*." *Music Theory & Analysis. International Journal of the Dutch-Flemish Society for Music Theory* no. 3:1-26.
Abstract: "In an influential 1981 article, "Interpreting an Arithmetical Error in Boethius's D*e institutione musica* (iii.14–16)," André Barbera drew attention to a problematic set of arithmetical proofs. At *Ins. mus*. 3.14ff. Boethius purports to prove that (1) the minor semitone is larger than three commas but small than four, (2) the major semitone is larger than four commas but smaller than five, and (3) the tone is larger than seven commas but smaller than eight. All of these conclusions are correct, but the mathematical procedure seems inherently flawed, for Boethius manipulates the numerical difference between the terms of a ratio as if it were an accurate quantification of the resultant interval.
Barbera maintained that the rationale motivating the erroneous mathematics "lies at the heart of Pythagorean cosmogony." Boethius (Barbera argued) set out to find the numerical truth underlying acoustic phenomena and "seems to have been satisfied by the apparent numerical verification of what he could hear." The origin of this "arithmetical error" can be specified with greater precision than the vague invocation of "Pythagorean cosmogony" allows. First, I demonstrate that Boethius's arithmetical error is not nearly as erroneous as Barbera and others would have us believe; rather, it represents an approximate method of calculation used to prove relationships otherwise incalculable. Secondly, I argue that this approximate method was not developed by Boethius but was faithfully translated from his immediate Greek source, Nicomachus of Gerasa's (now lost) *Eisagoge musike.* Thirdly, I suggest that this method, or at least its basic principle, was not independently developed by Nicomachus either, for similar arithmetical methods arose within the early stages of the Greek commentary tradition on Plato's *Timaeus*."

27. Jeserich, Philipp. 2013. *Musica Naturalis: Speculative Music Theory and Poetics, from Saint Augustine to the Late Middle Ages in France*. Baltimore: John Hopkins University Press.
Translated from the German *Musica naturalis*, Stuttgart: Franz Steiner, 2008 by Michael J. Curley and Steven Rendall.
Chapter 6: Boethius, *De institutione arithmetica* and *De institutione musica*, pp. 117-154.

28. Kárpáti, András. 1987. "Translation or Compilation? Contributions to the Analysis of Sources of Boethius' *De institutione musica*." *Studia Musicologica Academiae Scientiarum Hungaricae* no. 29:5-33.
I. To understand the *De institutione musica*, first we have to see clearly the intentions of the writer. Boethius has few Latin antecedants. Both Cassiodorus and he himself mention the work of the 4th century Albinus. (9) Albinus wrote a geometry and probably a dialectics, too. (10) Cassiodorus esteemed high of Gaudentius' work *Introductio harmonica* (11) and wrote that it was translated to Latin by his friend, Mutianus.12 It is not clear whether Boethius could know this or not. Besides these, all the musicological works written in Latin are a few Vitruvius-excursions, (13) the 9th book of Martianus Capella's illustrious *De nuptiis* and some chapters of *De die natali* by Censorinus. (14) Augustinus' *De musica* deals only with rhythmics and metrics, and we also know about Apuleius' work on music, (15)

though it is lost. So Boethius was right in thinking that there was no comprehensive work in Latin which could contain high-level theoretical matter and from which Greek music — theory could be thoroughly learnt. Especially the interpretation of the Pythagorean theory was missing, which used mathematical means, for Censorinus and Martianus Capella borrowed a lot from Aristeides Quintilianus' work, who belonged to Aristoxenus' school. So Boethius' intentions were clearly to supply this basic lack." (pp. 6-7)
(9) Cf. Cassiodorus, *Institutiones* 5, 10, p. 19. (Gerbert).
(10) " Albinus quoque de eisdem rebus scripsisse perhibetur, cuius ego geometricos quidem libros editos scio, de dialectica vero diu multumque quaesitos reperire non valui." (*De interpretatione ed. secunda* I. PL 64, 394 A).
(11) " Habetis hie (sc. in Bibliotheca Romae) Gaudentium Mutiani Latinum, quem si sollicita intentione relegitis, huius scientiae (sc. musicae) vobis atria patefacit." (Cass.: *Inst*. 5, 10.)
(12) Cf. Cassiodorus Ib.
(13) Vitruvius, *De architectura* 5, 5, 3.
(14) Censorinus, *De die natali* 10; 12; 13.
(15) Cf. Cassiodorus I. c. 1b

29. Kibre, Pearl. 1981. "The Boethian *De Institutione Arithmetica* and the Quadrivium in the Thirteenth Century University Milieu at Paris." In *Boethius and the Liberal Arts: A Collection of Essays*, edited by Masi, Michael, 67-80. Bern: Peter Lang.

30. Kijewska, Agnieszka. 2003. "Mathematics as a Preparation for Theology: Boethius, Eriugena, Thierry of Chartres." In *Boèce ou la chaîne des savoirs*, edited by Galonnier, Alain, 625-648. Louvain-Paris: Éditions Peeters.

31. Kirby, Helen. 1981. "The Scholar and his Public." In *Boethius: His Life, Thought and Influence*, edited by Gibson, Margaret, 44-69. Oxford: Blackwell.
"Boethius' motives in embarking upon his intellectual projects were not as grandiose as has sometimes been argued: from the evidence which remains he cannot be regarded as having conceived a 'Hellenist Renaissance' on a significant scale. He saw his role as one of enriching the Latin intellectual tradition with the transfusion of Greek material, in the mould of many Latin intellectuals before him. He believed that he had an audience, interested in Greek culture if not skilled in the Greek language, and the opposition to such an enterprise (if indeed contemporaries recognised his intellectual activities as having the coherence this word implies) should not be overemphasised. Boethius' intention in his philosophical programme was, and this cannot be overstressed, a highly technical one, that of reconciling the philosophical schemes of Plato and Aristotle, and he himself makes no claim for it to have any greater significance than this." (p. 59)

32. Lemoine, Fannie J. 1981. "The Precious Style as Heuristic Device: The Function of Introductions to the Arts in Martianus Capella and Boethius." In *Boethius and the Liberal Arts: A Collection of Essays*, edited by Masi, Michael, 51-65. Bern: Peter Lang.

33. Mair, John. 1981. "The Text of the *Opuscula Sacra*." In *Boethius and the Liberal Arts: A Collection of Essays*, edited by Masi, Michael, 206-213. Bern: Peter Lang.

34. Masi, Michael. 1971. "Manuscripts Containing the *De Musica* of Boethius." *Manuscripta. A Journal for Manuscript Research* no. 15:89-95.

35. ———. 1974. "Boethius and the Iconography of the Liberal Arts." *Latomus* no. 73:57-75.
"To anyone who reads the texts of the quadrivium, it is obvious that for Boethius proper order was very important.
(...)
Each discipline is logically prior to the following and each is dependent on the previous for its principles of procedure. The four arts of the quadrivium must be studied in the given order and they ultimately serve as preparation for a study of philosophy; the conclusion of their study is an approach to the wisdom of the

Consolation of Philosophy. Indeed, without the study of the quadrivium, true wisdom is not available to the student.
(...)
"The second point in my exposition, while extending the scope of the discussion, at the same time involves it in a simple but important philosophical consideration. Basic to the nature of the quadrivium is an understanding of the purpose of the arrangement of the disciplines. It is the matter of the nature, arrangement, and purpose of the disciplines where we see the increasing schematization of the allegories of the liberal arts during the late Middle Ages." (pp. 58-59 note momitted)
(...)
"At this point it is important to review the conceptual binding force within the quadrivium which enables its disciplines to be extended universally to all the arts and to the diverse modes of thought beyond them. The logical principles of the disciplines in the quadrivium are drawn from number theory, which is explained in considerable detail and with some application by Boethius in the *De Arithmetica*.
(...)
The proportional definitions of the *De Arithmetica* are extended to the relationships between sounds in the *De Musica*, second discipline of the quadrivium.
(...)
From the *De Musica*, the student of the liberal arts proceeded to the *De Geometria*. The science of geometry applies the concepts of proportion and order to the dimensions of planes and to the shapes of figures extended into solids. To these geometrical figures are applied the rules of harmonic proportion as outlined in the *De Arithmetica* and realized in sounds by the *De Musica*.
(...)
In the study of astronomy, the principles of order and proportion, the metaphor of harmony, and the ideals of the proper arrangement of parts receive their broadest extensions. Man was thought to embark on the highest order of learning when he undertook a survey of the heavens. In the context of the liberal arts, we find this larger meaning in the arrangement of celestial spheres in Dante's *Paradiso*." (pp. 65-66)

36. ———. 1981. "Boethius' *De Institutione Arithmetica* in the Context of Medieval Mathematics." In *Congresso Internazionale di Studi Boeziani (Pavia, 5-8 ottobre 1980): atti*, edited by Obertello, Luca, 263-272. Roma: Editrice Herder.
"During the last three or four centuries, the name of Boethius has been linked almost exclusively with the *De Consolatione Philosophiae*. However, as it has been recently recognized among Boethian scholars more generally, Boethius was recognized during the Middle Ages as the author of a variety of important works on logic, music, theology and mathematics (1). Some of these works became standard texts for the curriculum of the seven liberal arts. In recent years there has been considerable research into the writings and texts of Boethius as seen in the context of medieval culture (2). These more recent studies and re-evaluations of Boethius' work have been important because they demonstrate the complexity of Boethius' influence - notably that of his mathematics. His *De Arithmetica* had an influence not only on writers of mathematics but also on those who were concerned with the principles of music, literature, ethics, and architecture. In this essay I would like to examine the Boethian relationship with other medieval mathematical writing, and in so doing hopefully I might demonstrate his authoritative standing among writers of such texts. But, as well, in order to show the full scope of his work I would like to touch briefly on works outside the field of mathematics, although it will be possible to do so in only the most sketchy manner." (p. 263)
(1) See, for example, the entry under "Boezio", *Dizionario Biografico degli Italiani,* Voi. 11 (1969), 142-165.
(2) See Pearl Kibre, « The Quadrivium In Thirteenth Century Universities (with special reference to Paris) », in *Arts Libéraux au moyen-âge* (Montreal, Canada: 1969), pp. 175-191.

37. ———, ed. 1981. *Boethius and the Liberal Arts: A Collection of Essays*. Bern: Peter Lang.
Contents: Introduction 1; Myra L. Uhlfelder: The Role of the Liberal Arts in Boethius' *Consolatio* 17; Eleonore Stump: Boethius and Peter of Spain on the *Topics* 35; Fannie J. Lemoine: The Precious Style as Heuristic Device: The Function of Introductions to the Arts in Martianus Capella and Boethius 51; Pearl Kibre: The Boethian *De Institutione Arithmetica* and the *Quadrivium* in the Thirteenth Century University Milieu at Paris 67; Michael Masi: The Influence of Boethius *De Arithmetica* on Late Medieval Mathematics 81; Ubaldo Pizzani: The Influence of the *De institutione musica* of Boethius up to Gerbert D'Aurillac 97; Calvin M. Bower: The Role of Boethius's *De institutione musica* in the Speculative Tradition of Western Musical Thought 157; Julia Bolton Holloway: "The "Asse to the Harpe": Boethian Music in Chaucer 175; Menso Folkerts: The Importance of the Pseudo-Boethian *Geometria* during the Middle Age 187; Pierre Courcelle: Boethius, Lady Philosophy, and the Representations of the Muses 211-218.

38. ———. 1981. "The Influence of Boethius *De Arithmetica* on Late Medieval Mathematics." In *Boethius and the Liberal Arts: A Collection of Essays*, edited by Masi, Michael, 81-95. Bern: Peter Lang.
"In another essay of this collection, Professor Ubaldo Pizzani has made a study of the Boethian *De Musica* and how it was disseminated throughout Europe until the High Middle Ages. He has made clear that the history of the *De Music*a was closely tied with the spread of the *De Arithmetica* which seems to have been intended - or at least to have served in the medieval schools - as an introduction to the music theory. It is my intention in this chapter of the collection to extend Professor Pizzani's survey to the late history of the *De Arithmetica*. Wc should be able to see that the history of the Boethian mathematics underwent several interesting developments, most of these quite apart from its connection with the music theory. First and most significantly it must be noted that the Boethian arithmetic did not lose its importance after the influx of Arabic mathematics and the re-introduction of Greek number theory. If we are to judge from the number of extant manuscripts which contain the *De Arithmetica*, by the frequency of citations in other treatises (with or without the mention of Boethius' name) and by the number of early printed editions through the 16th century (at least 25), (1) we must conclude that the Boethian mathematics enjoyed an extraordinary increase in popularity and influence between 1200 and 1600.
That Boethius' mathematics should have become so widely used may seem surprising since the superior texts of Euclid, Nicomachus (Boethius' source) and Archimedes were available, could be read by many scholars, and were being translated. Moreover, as all students of the period are aware, the nature of mathematics was undergoing thorough and wide reaching changes at this time. The needs of a growing merchant trade which demanded efficient bookkeeping were responded to by an increasingly more sophisticated computational mathematics couched in the recently adapted Arabic number system. Though the new mathematical technique were initially slow to grow in European soil, by the 15th century hundreds of computational works (2) were available for those who wanted to learn. The ascendency of the Boethian mathematics in this context indicates a far more interesting aspect about the development of mathematics in the Middle Ages than that it was simply evolving into modem algebra and trigonometry. An examination of the various texts dealing with mathematics shows that this discipline was becoming highly diversified in nature by the late Middle Ages.
Until the late Middle Ages, a large portion of the mathematics studied in the schools and universités was a carry-over of earlier number theory, unoriginal and impractical. It was a mathematics which oriented the student to philosophical study and was imbued with the terminology of logic. As a preparation for higher philosophical study, it had once served its purpose well, but it had long since ceased to grow by the 15th century. But practical and computational mathematics slowly broke with the old number theory and began a new strain of mathematics. This

break occurred outside the universities and probably began very early in the Middle Ages. It was a new strain of mathematics that lived in the counting rooms of merchants and its greatest exponent was Fibonacci, the son of a trader. Certainly much of the computational mathematics was as servile to the merchant as the older number theory was to the philosopher. Some few thinkers, subtle and perceptive in their study of both Greek and Arabic numbers, such as Bradwardine, Nicholas of Oresme and Fibonacci, achieved a scope of mathematical vision not fully appreciated until recent times.
By the late Middle Ages the *De Arithmetica* had become moribund, and the widespread popularity of Boethius served, perhaps, to slow down the progress of mathematical innovation. The text of the *De Arithmetica* was inherited by the universities as a scrap from that vast learning of Greek thinkers and had become fossilized as part of the Liberal Arts curriculum, a once vital program of studies. But, for a small number of original thinkers (whose proportion among mathematicians has perhaps remained a constant even to our own times) Boethian definitions of numbers, the classification of number and ratio, and the definitions of the types of proportionalities were the starting points for new understandings.
I have accordingly divided my study into two parts. Initially I will survey the evidence which makes the Boethian treatise the best known mathematical work of the Middle Ages. Without attempting to evaluate the works cited, I will review a few treatises which show the way in which Boethius was adapted. When the *De Arithmetica* was not used directly as a text, it appeared in shortened form (an epitome) or merely excerpted to reduce its length. In the second part of the essay I will attempt to demonstrate how Boethian ideas provided seeds for more original thought in the works of a small number of innovative mathematicians. The most important of these are Thomas Bradwardine, Albert of Saxony (whom he influenced), Roger Bacon and Nicholas of Oresme." (pp. 81-82)
(1) In the introduction to my translation of the *De Arithmetica* (to be published soon by Rodopi, Amsterdam [1983]), I have listed almost 200 manuscripts of the *De Arithmetica* extant. A list of printed editions may be found in David Eugene Smith, *Rara Arithmetica* (Boston: Ginn & Co., 1908), p. 27.
(2) Descriptions of many such works may be found in Smith's *Rara Arithmetica*.

39. McCluskey, Stephen C. 2012. "Boethius's Astronomy and Cosmology." In *A Companion to Boethius in the Middle Ages*, edited by Kaylor Jr., Noel Harold and Phillips, Philip Edward, 47-73. Leiden: Brill.

40. McNamee, Megan C. 2022. "Grid Space in Boethius's *De institutione arithmetica*." In *The Diagram as Paradigm. Cross-Cultural Approaches*, edited by Hamburger, Jeffrey F., Roxburgh, David J. and Safran, Linda, 389-418. Washington, DC: Dumbarton Oaks Research Library and Collection.

41. Moyer, Ann E. 2002. "The demise of the quadrivium and the beginning of the Scientific Revolution: Boethius in the sixteenth century." *Intellectual News* no. 10:69-77.

42. ———. 2012. "The Quadrivium and the Decline of Boethian Influence." In *A Companion to Boethius in the Middle Ages*, edited by Kaylor Jr., Noel Harold and Phillips, Philip Edward, 479-517. Leiden: Brill.

43. Müller, Kathrin. 2012. "Theorie und Materialisierung der Zahl in Boethius' *De institutione arithmetica*." In *Was Zählt. Ordnungsangebote, Gebrauchsformen und Erfahrungsmodalitäten des "numerus" im Mittelalter*, edited by Wedell, Moritz, 81-102. Köln: Böhlau Verlag.

44. Otisk, Marek. 2022. "The Definitions of Number in Boethius's *Introduction to Arithmetic*." *Philosophical Readings* no. 14:16-26.
Abstract: "The paper enquires into the reasons why Boethius altered the passage addressing the definition of number in his loose translation of I*ntroduction to Arithmetic* by the Neopythagorean philosopher and mathematician Nicomachus of Gerasa. While Nicomachus's text contains three definitions of number, Boethius

lists only two. However, he also pays attention to the definition he omits, even though he does not regard it as a proper definition.
In his view it fails to embody the essence of number, and is to be understood as a description of the components constitutive of the substance of number. Although this is a possible explanation of Boethius's dismissal of the definition provided by Nicomachus, the description also occupies an important place in relation to the general characteristic of number, because Nicomachus's definitions fully correspond to the three basic topics which were central to contemporary arithmetic, viz. the science of number: number as discrete quantity, referring to the properties of numbers and their classifications; number as collection of units, leading to the topic of figural numbers; and number as quantity emanating from unit and subsequently returning to it, corresponding with numerical ratios, sequences and their transfers."
Note 11: "Nicomachus in his Introduction to Arithmetic lists three definitions of number:
(1) number as discrete quantity (πλήθος ὡρισμένον);
(2) number as collection of units (μονάδων σύστημα);
(3) number as quantity emanating from unit (ποσότητος χύμα ἐκ μονάδων συγκείμενον).
Nicomachus, I*ntr. arith.* I, 7, 3, p. 13,7–8. English translation by M. L. D'Ooge (N*icomachus of Gerasa, Introduction to Arithmeti*c. Transl. M. L. D'Ooge. New York – London: Macmillan, 1926, p. 190)

45. Palisca, Claude V. 1990. "Boethius in the Renaissance." In *Music Theory and its Sources: Antiquity and the Middle Ages*, edited by Barbera, André, 259-280. Notre Dame: University of Notre Dame Press.
Reprinted in: C. V. Palisca, *Studies in the History of Italian Music and Music Theory*, Oxford: Clarendon Press, 1994, pp. 168-188.

46. Panti, Cecilia. 2017. "Boethius and Ptolemy on Harmony, Harmonics and Human Music." *Micrologus*:3-36.
Abstract: "The contribution analyzes how Boethius elaborates the concepts of *vis* or fac*ultas armonicae* (power of the harmonics) and *vis armonica* (harmonic power), both derived from Ptolemy's *dynamis harmonike*, namely the power to grasp the distinctions between high and low sounds and produce an immediate judgment of pleasure or distrust. The essay aims to demonstrate that this concept provides the theoretical equipment for the study of the Boethian second genus of music, the *musica humana*, and consequently that Boethius' famous threefold division of music (*instrumentalis, humana* and *mundana*) implies a special methodological approach for each area. In particular, with the unfinished book 5 of the *De institutione musica*, Boethius turns to the practical usefulness of music, and shifts from a pure mathematical science, the *instrumentalis*, to an intermediate one, the *humana*, which involves also ethics and physics. For Boethius, this second kind of music concerns human wellness and good government, and its knowledge is particularly recommended to rulers and those who have political responsibilities."

47. ———. 2018. "Beauty, Ethics and Numbers in Boethius' Quadrivial Treatises." *Aisthesis* no. 11:67-79.
Abstract: "The convergence of the Neoplatonic/Neopythagorean approach with the Aristotelian organization of the sciences is one of the most interesting features that characterizes the two influential mathematical treatises on On Arithmetics (*De institutione arithmetica*) and On Music (*De institutione musica*) by Severinus Boethius. Basing his reasoning on Nicomachus and Ptolemy, Boethius follows the philosophical tradition that had tried to reconcile Plato's and Aristotle's views. This attitude is examined in the present paper as regards Boethius' response concerning the relation between numbers, ethics and aesthetics. His view emerges as coming out of a rather complex construction, which assigns the ethical scope of mathematics in indicating to the human mind how to correct the ratios that realize the best relationship in movements of the soul and the body. More precisely, its ethical aim is to correct the specific form of movement of human beings, that is

their actions, exemplified in the way in which mathematical ratios represent the forms of government and musical ratios evoke and heal psychophysical affections. More complex, on the other hand, is the relationship between mathematics and beauty. In clear antithesis to the position taken by Augustine on the beauty of the rhythmic patterns that better represent the beauty of unity,
Boethius does not relate the mathematical ratios of the consonances to an esthetical judgment by making use of the category of beauty. For him, the physical world is totally immersed in changes and movements, and this cannot but impede things from expressing the stable unity, which is required for contemplating the beautiful."

48. Pingree, David. 1981. "Boethius' Geometry and Astronomy." In *Boethius: His Life, Thought and Influence*, edited by Gibson, Margaret, 155-161. Oxford: Blackwell.
"Cassiodorus in a letter he wrote to Boethius on behalf of Theoderic in about 507, (1) attributes to his young correspondent translations of Pythagoras on music, of Ptolemy on astronomy (for which see below), of Nicomachus on arithmetic, and of Euclid on geometry; later on, in the Institutiones (2), he refers again to Boethius' translation of Euclid." (p. 155)
(...)
"It has been argued that only Boethius could and did translate Euclid from the Greek before the twelfth century. However, this cannot be stated with certainty, for it is clear from the Praeceptum Canonis Ptolemei discussed below that reasonably competent Latin translations of Greek works on the exact sciences were still being made in the sixth century, after Boethius' death. There survive, in fact, two fragments of quite different translations of Euclid that cannot be demonstrated to have anything to do with either Boethius or with M, though one is connected with Luxeuil, the other with Corbie." (p. 156)
(...)
"Cassiodorus in his letter to Boethius, as we have seen, attributes to the latter a translation of Ptolemy on astronomy. In the Institutiones (29) he gives a description of the contents of Ptolemy's *Canones* which corresponds quite well with the *Praeceptum Canonis Ptolemei* which survives in some eight manuscripts copied between the late tenth and early thirteenth centuries. (30) Naturally, these references by Cassiodorus raise the possibility that the *Praeceptum* is the translation made by Boethius. This, however, cannot be the case. For the *Praeceptum*, which comprises two reasonably competent translations of instructions for the use of the Ptolemaic *Handy Tables*, mentions the year 535 as current. (31) It was, perhaps, intended for use in the Christian school that Cassiodorus and Pope Agapetus hoped to establish in Rome in just that year or the next. (32) Its existence proves that Boethius was not the only translator of Greek texts belonging to the quadrivium in the late fifth and early sixth centuries — a conclusion toward which the Verona fragments of Euclid seem also to point. Thereby the argument that M most probably represents Boethius' translation because we know of no other translator (33) loses some of its force, though Boethius certainly remains the chief suspect." (p. 159)
(1) *Variae* I. xlv. 4, ed. T. Mommsen (Berlin, 1894: *MGH* [*Monumenta Germaniae Historica*] Auct. Ant. xii), p. 40. On the translations of Euclid's *Elements* see J. Murdoch in *Dictionary of Scientific Biography* (New York, 1971), iv. 437-59.
(2) *Institutiones* II. vi. 3, ed. R. A. B. Mynors (Oxford, 1937), p. 152.
(30) An edition is being prepared by N. Swerdlow and the present writer.
(31) *Praeceptum* II. 1: a Diocletiano usque nunc anni sunt CCLI; cf. II. 11. In *Praeceptum* I. 23 is mentioned Augustus 383=ad 354.
(32) *Inst*. I praef. 1, ed. cit. (note 2 above), p. 3. Agapetus was pope from 13 May 535 till 22 April 536.
(33) Folkerts, [*Boethius' Geometrie II. Ein mathematisches Lehrbuch des Mittelalters*, Wiesbaden, 1970] p. 72.

49. Pizzani, Ubaldo. 1981. "The Influence of the *De Institutione Musica* of Boethius up to Gerbert D'Aurillac: A Tentative Contribution." In *Boethius and the Liberal Arts: A Collection of Essays*, edited by Masi, Michael, 97-156. Bern: Peter Lang.

"Among those writings of Boethius which transmitted to the Middle Ages so much of the ancient culture, the *De Institutione Musica* must certainly occupy a place of special prominence. The second treatise of the quadrivium, according to the outline apparently sketched by Boethius himself in the first chapter of the *De Institutione Arithmetica* (1) its fame is attested not only by the great number of manuscripts in which it has reached us, (2) but also (and above all) by the imposing mass of scholia - largely unpublished (3) -preserved in those manuscripts, as well as by the flourishing medieval musicological production that found inspiration in Boethius as the most learned and reliable authority on the ancient musical culture. (4)

It is mainly to this last aspect that scholars as a rule have directed their attention, (5) owing partly to the fact that Boethius, with his approximations and misunderstandings, did not always have a positive and stimulating effect on the musical theories of the Middle Ages. (6) But there is another factor, no less important, in the continuous good fortune of this treatise: its influence, together with that of the *De Institutione Arithmetica*, and the lost treatises *De Institutione Geometrica* and *De Institutione Astronomica*, (7) on the didactic applications of the quadrivium. Here the medieval schools did not have in Boethius their exclusive and unchallenged master, but turned, as well, to the great syntheses of such teachers as Martianus Capella, Cassiodorus, and Isidor of Seville, not to mention the *De Musica* of St. Augustine. Nevertheless, there is no doubt that the Boethian corpus, with its greater amplitude, and with the prestige conferred upon its author by his logical and theological works, played a decisive and preeminent role. This is true especially in the Carolingian (8) and post-Carolingian periods, whence derive most of the codices bearing the treatises in question. (9) It is more difficult to determine the role they played in the nearly three centuries intervening between their composition and the Renaissance prompted by Charlemagne.

The fact is that the influence of Boethius on the authors of that period - particularly Cassiodorus and Isidor of Seville, who both dealt with the artes of the quadrivium - is not conclusively demonstrable, particularly as far as music is concerned. An exceptional case is that of the Venerable Bede, under whose name we have a *Musica Theorica* that is definitely linked to Boethius, but which, as I have demonstrated elsewhere (10) and will illustrate later in this essay, came into being under very particular circumstances that do not allow us to attribute it to the Monk of Jarrow *sic et simpliciter*. An investigation into this matter, culminating, as we shall see, in a proposal for a new form of edition for that unique work, will give us an opportunity to survey the mass of scholia relating to the *De Institutione Musica* (for, indeed, the *disiecta membra* of the so-called *Musica Theorica* are nothing more than scholia); to examine some of the complex problems attending the preparation of such an edition and the identification of sources; and lastly, to ascertain the potential role of these scholia in clarifying in what forms and ways the Boethian text was used in medieval scholastic activity. At that point, with a brief survey of the coeval musicological production, we shall bring our investigation to a close. Our purpose is not so much to obtain definitive solutions to individual problems as to define their terms and to establish the areas of research and the methodological criteria while trying to trace an outline - however tentative and problematical - of the fortune of the *De Institutione Musica* from Boethius to the scholiastic activity of Gerbert d'Aurillac. In the absence of a thorough documentation, it is the history up to the 9th century, at least, that will prove hard to grasp in even its most essential stages. But the problems that it poses, whether in philology, strictly speaking, or in the history of culture, cannot be evaded by anyone who wishes to understand through what adventures the *De Institutione Musica* passed so decisively into the scholastic and musicological traditions of the Middle Ages - after a period, as it would appear, of total eclipse." (pp. 97-98)

Note: the editor is grateful to Mr. Peter Gimpel who translated this essay from the Italian, working in close collaboration with the author.

(1) *De Institutione Arithmetica* I, 1. All citations from the Boethian treatises on arithmetic and music are taken from the edition of G. Friedlein (Leipzig, 1867; reprinted, 1966) and cited hereafter as DIA and DIM.

(2) A nearly complete list has been furnished recently by M. Masi, "Manuscripts Containing the *De Musica* of Boethius," *Manuscripta*, 15 (1971), 89-95 and "A Newberry Diagram of the Liberal Arts," *Gesta*, XI/2 (1973), 56.
(3) As far as I know, the only scholia to have yet been published are two by Gerbert d'Aurillac (N. Bubnov, *Gerberti Opera Mathematica* [Berolini, 1899] pp. 28-31), one on the harmony of the spheres (R. Bragard, "L'harmonie des sphères selon Boèce," *Speculum*, 4 [1929], pp. 206-13, and some others in my article, "Uno pseudo-trattato dello pseudo-Beda," (*Maia*, I [1957], 36-48). It should be noted, however, that a substantial mass of scholia has been packed into this so-called treatise attributed to the Venerable Bede, as we shall see in the course of this study.
(4) The texts are assembled in good part in the fundamental though somewhat obsolete collections of M. Gerbert, *Scriptores Ecclesiastici de Musica Sacra Potissimum* (Typis San Blasianis, 1784; reprinted, 1931) and E. Coussemaker, *Scriptores de Musica Medii Aevi,* 4 vols. (Paris, 1864-76). More up-to-date on the critical and textual level is the *Corpus Scriptorum de Musica*, but the latter is still far from completion.
(5) See the ample bibliography after the article on Boethius by R. Wagner in the great German encyclopedia, *Die Musik in Geschichte und Gegenwart*, Band II (Kassel und Basel, 1952), col. 49-57.
(6) See for the bibliographical references the above-cited article by R. Wagner, coll. 54-55; see also my observations in "Studi sulle fonti del De Institutione Musica di Boezio," *Sacris Erudiri*, 16 (1965), 87, 128 ff.
(7) That Boethius completed the entire cycle of the quadrivium is deducted from incontestable evidence. (See the article, "Severino Boezio" in the *Dizionario Biografico degli Italiani*, 11 [Roma, 1969], 142-65.)
(8) Our most ancient voice, in that sense, is perhaps that of Aurélien de Moutier-St. Jean (Aurelianus Reomensis), who lived in the first half of the 9th century and was the author of a *Musica Disciplina,* largely inspired by Boethius, whom he cites and exalts as *vir doctissimus* (p. 41, Gerbert) and *eruditissimus* (Gerbert, p. 32).
(9) None of the extant manuscripts of the *De Institutione Musica* dates from before the 9th century.
(19) U. Pizzani, "Uno pseudo-trattato."

50. Potiron, Henri. 1961. *Boèce: théoricien de la musique grecque*. Paris: Bloud et Gay.

51. Rimpler, Mark T. 2012. "The Enduring Legacy of Boethian Harmony." In *A Companion to Boethius in the Middle Ages*, edited by Kaylor Jr., Noel Harold and Phillips, Philip Edward, 447-478. Leiden: Brill.

52. Schrade, Leo. 1947. "Music in the Philosophy of Boethius." *The Musical Quarterly* no. 33:188-200.
"In the doctrine of music that Boethius formulated in his youth two elements, both of ethical nature, converge, and in this conjunction the ethical value of music surpasses that of any other discipline in the Quadrivium. For music as the art of sound exerts in all
events and by its very nature an influence upon the moral state of man, or, in the words of Boethius himself , music is capable of " improving or degrading the morals of men" . In addition to this, however, music as part of mathematics shares in those educational
ethics that are inherent in the disciplines of the Quadrivium. It contributes to the training of the intellect, which in the end must be totally free from all bodily impediments. This is the meaning of the education in which music assists in liberating the human mind. The music Boethius described at the beginning of his literary activity is of Platonic-Pythagorean origin. It has no direct contact with the Aristotelian system of philosophy. Music stands before philosophy; and the student of music is driven by the ethical impulse to learn how to benefit intellectually from the instrument that holds the key to the " purer reason of the mind" in philosophy. With the assumption of a pre-philosophic position of music, with the thesis of its ethical function in the process of education, and finally with the denial that music as

a “ science” could be part of philosophy proper, Boethius gives evidence that he wrote his works on the Quadrivium essentially as a Platonist. In it he had no intention — and no need— of reconciling the Aristotelian and Platonic schools of thought with each other.
(...)
It may be puzzling to find that an incomplete book on music, written by a man of only twenty, exercised the most extraordinary influence upon centuries to come. It is less puzzling when we take into account both the ethical function and the position music was given in relation to philosophy. And it seems to be very characteristic of this school of thought that many a later philosopher actually did what Boethius had done, that is to say, started any work in philosophy with a treatise on music as a primary necessity, without ever returning to music again. This procedure is by no means accidental; it bears all the marks of the situation in which Boethius’ work on music originated. The significance of this fact has been completely overlooked. Let us think of Augustine, to name only one author of the Latin world of humanities. The first work Augustine wrote is his treatise on music which, however, he did not complete in his youth. Though in later years he returned to the subject of music— in his commentaries on the Psalms— he did so merely for reasons of a religious nature which had nothing to do with the Musica as a discipline of the Quadrivium. When investigating the work of philosophers through the centuries we are surprised how often we find music opening the course of philosophical studies. Even Descartes, in 1618, still begins with an *Essay on Algebra* and the *Compendium of Music*. The theorists of music proper, also, in antiquity, the Middle Ages, through the 16th century (e.g., Glareanus) often first presented an “ introduction” to music. But the reason for this would require a special discussion." (pp. 193-194)

53. Stone-Davis, Férdia J. 2011. *Musical Beauty: Negotiating the Boundary between Subject and Object*. Eugene (OR): Cascad Books.
Contents: Acknowledgments IX; Introduction XI-XVII; 1. The Bocthian Understanding of the World, the Role of Beauty, and the Value of Music 1; 2. Inhabiting Harmony — The World as a Series of Relations: An Examination of the *Fundamentals of Music* 12; 3. Know Thyself — The Place of Humankind within Created Beauty: An Examination of the *Consolation of Philosophy* 34; 4 The Kantian Understanding of the World, the Role of Beauty, and the Value of Music 79; 5. The Play of Harmony — The Subjective Powers in Relation: An Examination of the *Critique of Judgment* 90; 6. Creating Beauty: Genius and the Work of Art 119; 7. Musical Beauty: An Enchanted Mode of Attention 158; Conclusion 191; Bibliography 195; Index 207.
"The nature of musical meaning, considered through its physicality, is also often clouded by frameworks built upon aesthetic categories and principles. These provide a lens through which the arts in general arc viewed. Such broad-sweep approaches tend towards a certain homogenization across their range and thereby result in distortion within accounts of individual arts. The uniqueness of the arts is not fully attended to. The specific category that will provide the focus for the task at hand is beauty. Beauty features within current aesthetic and theological discussion (standing more peripherally within the domain of philosophical aesthetics). Historically, however, it has been a dominant concept, acting as a cipher for underlying presuppositions that themselves comprise broader frameworks, both philosophical and theological.
It is on this basis that the two central figures of our narrative present themselves: Anicius Manlius Severinus Boethius (c.480-c.525) and Immanuel Kant (1724-1804)." (p. XIV)
(...)
"Initially, then, we will start with Boethius for whom the world is knowable and for whom, as a result, the physical world is of value. Here, within an integrally theological framework, beauty is understood as harmony and as such is constitutive of the world: it is the principle by which the world coheres as a whole and a property of the material world. I will show how granting beauty this objectivity

allows it a cosmic meaning or “resonance” which extends both between and beyond subjects. I will then demonstrate how the Bocthian account of music illustrates his understanding of the material world and beauty. Considering beauty as the principle of harmony grants music significance in relation to both the intellectual and the material for, as physical sensation, music offers knowledge of the world. However, I shall show that ultimately Boethius stresses the intellectual to the detriment of the material, using the physical experience of music as merely a stepping-stone to intellectual perception through form (with form finding its ideal location in God). By virtue of the satisfaction and pleasure imparted by musics physicality, Boethius’ attention is re-invigorated and he is encouraged to re-focus on the world and, specifically, himself as part of the world. Ultimately, however, musical indeterminacy gives way to and is surpassed by the conceptual truths of reason." (p. XVI)

54. Stump, Eleonore. 1981. "Boethius and Peter of Spain on the Topics." In *Boethius and the Liberal Arts: A Collection of Essays*, edited by Masi, Michael, 35-50. Bern: Peter Lang.

55. Thompson, Jeremy. 2020. "An Early Lemmatic Commentary on Boethius’s *De institutione arithmetica*." *Archives d'histoire doctrinale et littéraire du Moyen Âge* no. 87:115-200.
Abstract: "This edition and study present the text of an early lemmatic commentary on Boethius's *De institutione arithmetica* 1, 1-5. The text bears the title *De initio A. M. S. Boetii pauca in arithmetica* and is newly reconstituted from the membra disiecta of a codex dating to the last quarter of the ninth century : Vatican, BAV, Reg. lat. 1424, f. 95r-98v and Leiden, University Library, VLO 88, f. 2r-10v. This commentary represents a fresh and significant witness to Boethius's ninth-century reception. It contains notes on his life, a treatment of grammatical and logical categories, and an apt handling of arithmetical topics broached in the main work. Ninth-century marginal annotations reveal an engaged reader perhaps influenced by Eriugenian thought."

56. Uhlfelder, Myra L. 1981. "The Role of the Liberal Arts in Boethius' *Consolatio*." In *Boethius and the Liberal Arts: A Collection of Essays*, edited by Masi, Michael, 17-34. Bern: Peter Lang.
"At the close of the Consolatio, God is looking down upon the cosmos while the newly liberated and restored Boethius, with the recovery of *mens* at the end of his philosophical return, is looking up as far as possible toward the Divine Light, who judges His well-ordered world and perceives everything though He Himself is beyond understanding. The whole situation is now the diametrical opposite of what it had been when Boethius, at the beginning, was indulging his grief with his elegiac Muses.
The preceding example is interwoven with the important judicial metaphor in the *Consolatio*. In passing judgment on God, Boethius had been influenced at the beginning by his personal experience of suffering shameful injustice at the hands if men, experience of which the autobiographical material in Book 1 provides ample evidence. By the end, however, Boethius has ascended beyond the limits of reason to accept on faith the goodness of the Divine Judge whom, even in his affliction, he had continued to recognize as Creator and Ruler (I. P.6. 4).
lu bringing about the changes so dramatically portrayed in these opening and closing scenes, the Liberal Arts make a contribution which it is the main purpose of this paper to examine." (p. 19)

57. Walden, Daniel K. S. 2015. "Charting Boethius: music andf the diagrammatic tree in the Cambridge University Library's *De institutione arithmetica*, MS II. 3. 12." *Early Music History* no. 34:207-228.
Abstract: "This article discusses a full-page schematic diagram contained in a twelfth-century manuscript of Boethius’ *De institutione arithmetica* and *De institutione musica* from Christ Church Cathedral, Canterbury (Cambridge University Library MS Ii.3.12), which

has not yet been the subject of any significant musicological study despite its remarkable scope and comprehensiveness. This diagrammatic tree, or arbor, maps the precepts of the first book of *De institutione arithmetica* into a unified whole, depicting the ways music and arithmetic are interrelated as sub-branches of the *quadrivium*. I suggest that this schematic diagram served not only as a conceptual and interpretative device for the scribe working through Boethius' complex theoretical material, but also as a mnemonic guide to assist the medieval pedagogue wishing to instruct students in the mathematics of *musica speculativa*. The diagram constitutes a fully developed theoretical exercise in its own right, while also demonstrating the roles Boethian philosophy and mathematics played in twelfth century musical scholarship."

58. White, Alison. 1981. "Boethius in the Medieval Quadrivium." In *Boethius: His Life, Thought and Influence*, edited by Gibson, Margaret, 162-204. Oxford: Blackwell.
"The quadrivium' designated the mathematical arts of arithmetic, music, geometry and astronomy. (2) Boethius appears to have been the first writer to use the term to delineate these areas of knowledge existing in the timeless world of being. For these arts were seen not merely as isolated sources of skills and information, but as an indispensable path to abstract knowledge. The created world, Boethius says, appears to have been formed according to number, 'for this was the principal design in the mind of the Creator'. (3) So the arts which consider number in its pure or solid form, static or mobile, provide paths through the creation to its source in the incorporeal world wherein lies true wisdom. This was the direction of Christian contemplation as much as Neoplatonic. It found a classic source in the verse 'Thou hast ordered all things in number, measure and weight' (*Wisd*. 11:21). Knowledge of the rational, numerical structure of the universe would lead to knowledge of the divine nature and to apprehension of God himself. (4)
Although Boethius set his mathematical works in the framework of Neoplatonic and Christian concepts of true wisdom, his immediate aim was severely practical: to make available to the Latin world the treasures of Greek learning. His *De Arithmetica* is chiefly a translation of a treatise by Nicomachus of Gerasa (2nd cent a.d.); his music draws on several Greek sources. (5) It seems likely too that Boethius made a translation of part of Euclid's *Elements* which was incorporated into medieval works bearing Boethius' name. (6) But the complex history of its transmission is largely independent of that of the *De Arithmetica* and *De Musica*, and it cannot be considered a definable, independent Boethian quadrivial text. It therefore lies outside the scope of this study. There is no surviving work by Boethius on astronomy; some scholars have pointed to possible traces of such a treatise, but the evidence is at best inconclusive. (7)
Boethius' approach to arithmetic and music was speculative and mathematical. 'Arithmetica' was the science of number, not calculation; 'musica' was harmonic theory based on Pythagorean mathematics of proportion, not music-making. This fact was firmly pointed out in the eleventh century by Guido of Arezzo, who declared that Boethius' *De Musica* was useful not to musicians, but only to philosophers. (8) But this is one of the cardinal reasons for the durability of his treatises: the science of number and proportion could serve as the foundation of diverse practical applications as long as the fundamental assumptions on which they were based were not challenged. Throughout the Middle Ages, Boethius' *De Arithmetica* and *De Musica* remained standard texts for these two arts of the quadrivium, sometimes supplemented by other medieval works, but never ousted from the canon." (pp. 162-163)
(2) On the liberal arts, see: ed. J. Koch, *'Artes Liberales' von der Antiken Bildung zur Wissenschaft des Mittelalters* (Leiden, 1959), but see review by C. Leonardi in *Studi Medievali*, 3 ser., II (1961), 268-75; H. Marrou, *Histoire de l'éducation dans l'antiquité* 6 edn (Paris, 1965) ; idem, 'Les arts libéraux dans l'antiquité classique', *Arts libéraux et philosophie au Moyen Age, Actes du IVe Congrès International de Philosophie Médiévale* (Montreal/ Paris, 1969), pp. 5-27. On the quadrivium, see: M. F. Bukofskcr, 'Speculative thinking in mediaeval music', *Speculum* XVII (1942),

165 180; '*Artes Liberales*', ed. Koch (1959), pp. 1-31 ; 107-23; E. A. Lippmann, 'The place of'music in the liberal arts', *Aspects of Mediaeval and Renaissance Music, A Birthday Offering to G. Reese*, ed. J. La Rue (London, 1967), pp. 545-59; W. H. Stahl, *Martianus Capella and the Seven Liberal Arts* (New York, 1971), I. 3-70; 125-227; L. Obertello, 'Boezio, le scienze del quadrivio e la cultura medievale', *Atti dell'Acad. Ligure di sc. e lett.* xxviii (1971), 152-70; G. Beaujouan, 'L'enseignement du "Quadrivium" ', *Settimane di Studio* XIX. ii (Spoleto, 1972), 639-67; I have not yet seen B. Münxelhaus, *Pythagoras Musicus: Zur Reception der Pythagoreischen Musiktheorie als quadrivialer Wissenschaft im lateinischen Mittelalter* (Bonn/Bad Godesberg, 1976).
(3) Boethius, *De Arithmetica* I. 2: ed. cit., [Friedlein] p. 12, lines 14-17.
(4) Cf. Augustine, *De Libero Arbitrio* II. xvi-xvii. 41-6; L. Schrade, 'Die Stellung der Musik in der Philosophie des Boethius als Grundlage der ontologischer Musikerziehung', *AGP [Archiv für Geschichte der Philosophie*] XLI. iii (1932), 368-400.
(5) H. Potiron, *Boèce, théoricien de la musique grecque* (Paris, 1961) ; J. Caldwell, pp. 135-54 above [The *De Institutione Arithmetica* and the *De Institutione Musica*].
(6) Ps-Boethius, *Geometria* (ed. Friedlein [1867]) pp. 372-428; ed. M. Folkerts, *'Boethius' Geometrie II* (Wiesbaden, 1970); H. Weissenborn 'Die Boethius-Frage', *Abhandlungen zur Geschichte der Mathematik* ii (1879), 187-240; C. Thulin, 'Zur Überlieferungsgeschichte des Corpus Agrimensorum', *Göteborgs Kungl. Vetenskaps-och Vitterhets-saml*. xiv (1911), 3-68; B. L. Ullman, 'Geometry in the mediaeval quadrivium', *Studi di Bibliografia e di storia . . . Tammaro di Marinis* (Verona, 1964), iv. 263-85; D. Pingree, pp. 155-61 above [Boethius' Geometry and Astronomy].
(7) W. H. Stahl (op. cit., note 2 above, p. 173 and n. 6) and B. L. Ullman (art. cit. note 6 above, 278) suggested that it might be found in Naples, Bibi. Naz., V. A. 13 (s. x), but the Naples catalogue (Naples, 1827), p. 258, and N. Bubnov, *Gerberti Opera Mathematica* (Berlin, 1899), pp. 478-9, ascribe the material to Isidore, Cassiodorus and Bede. The Bobbio catalogue of the tenth century includes 'Libros Boetii III de aritmetica' (nos 384-6) 'et alterum de astronomia' (no. 387) ; the twelfth-century catalogue of St Bertin mentions 'Boetius de geometria et astronomia'. See G. Becker, *Catalogi Bibliothecarum Antiqui* (Bonn, 1885), pp. 69 and 182. But the 'astronomiae' are not explicitly identified as Boethius' works.

59. Woodcock, Edith. 1943. "The Influence of Boethius on Musical Thought." *Bulletin of the American Musicological Society*:30-32.